Problems for exercise – Gauss's method for systems of linear equations

1) Solve on your own the systems using Gauss's method with a chosen pivot element and without a chosen pivot element. Work with simple fractions:

a)
$$\left|\begin{array}{rrrr} 10x_1 & -7x_2 & & =7 \\ -3x_1 & +2x_2 & +6x_3 & =4 \\ 5x_1 & -x_2 & +5x_3 & =6 \end{array}\right.$$

Answer: (0,-1,1)

b)
$$\left|\begin{array}{rrrr} x_1 & +2x_2 & +3x_3 & =7 \\ -2x_1 & -4x_2 & -5x_3 & =-12 \\ 3x_1 & +5x_2 & +6x_3 & =15 \end{array}\right.$$

Answer: (1, 0, 2)

c)
$$\left|\begin{array}{rrrrr} x_1 & +2x_2 & -3x_3 & +x_4 & =-6 \\ -x_1 & -x_2 & +2x_3 & & =3 \\ 2x_1 & +x_2 & +2x_3 & -x_4 & =12 \\ & x_2 & -x_3 & +2x_4 & =-4 \end{array}\right.$$

Answer: (2, 1, 3, -1)

Commentary: If you worked conscientiously you will notice that in both cases (with or without a chosen pivot element) you will get **the same exact** results.

2) Solve the system using Gauss's method by using a calculating device

$$\left|\begin{array}{rrrr} 2{,}4759x_1 & +1{,}6235x_2 & +4{,}6231x_3 & = \quad 0{,}0647 \\ 1{,}4725x_1 & +0{,}9589x_2 & -1{,}3253x_3 & = \quad 1{,}0475 \\ 2{,}6951x_1 & +2{,}8965x_2 & -1{,}4794x_3 & = -0{,}6789 \end{array}\right.$$

а) without choosing a pivot element;

б) choosing a pivot element.

In both cases do the calculations with four digits after the decimal comma.

Answer:

If working without a chosen pivot element you will get:

$x_3 = -0{,}2443$, $x_2 = -2{,}0532$, $x_1 = 1{,}8286$.

If working with a chosen pivot element you will get:

$x_3 = -0{,}2442$, $x_2 = -2{,}0716$, $x_1 = 1{,}8405$.

Why are the answers different? Which answer is better?

3) A system is given:

$$\left|\begin{array}{rcrcrcl} 0{,}05x_1 & + & 0{,}07x_2 & + & 0{,}06x_3 & + & 0{,}05x_4 = 0{,}23 \\ 0{,}07x_1 & + & 0{,}10x_2 & + & 0{,}08x_3 & + & 0{,}07x_4 = 0{,}32 \\ 0{,}06x_1 & + & 0{,}08x_2 & + & 0{,}10x_3 & + & 0{,}09x_4 = 0{,}33 \\ 0{,}05x_1 & + & 0{,}07x_2 & + & 0{,}09x_3 & + & 0{,}10x_4 = 0{,}31 \end{array}\right.$$

The exact solution of this system is (1, 1, 1, 1).

Solve the system using Gauss's method:

а) without choosing a pivot element;

б) with a chosen pivot element from a column, row or from the entire matrix.

Keep four decimal digits in the calculations. How can you explain your results?

By Iliya Makrelov, ilmak@uni-plovdiv.bg